Ultra-wideband Radio Aided Carrier Phase Ambiguity Resolution in Real-Time Kinematic GPS Relative Positioning

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Biography

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Abstract

In this research, ultra-wideband radios (UWBs) are integrated into the real-time kinematic (RTK) algorithm using differential GPS techniques to achieve a highly precise relative positioning vector between GPS antennas. This has potential applications including an autonomous leader-follower scenario or an unmanned aerial refueling scenario. The UWBs give a range measurement between antennas, while the RTK solution gives a three dimensional relative positioning vector. This UWB range measurement can be integrated into the RTK algorithm to add robustness and increase accuracy.

When two GPS receivers are within a close proximity, most of the errors that degrade the GPS signal are correlated between the two receivers and can be mitigated by using differential techniques. This can be done using either a static base station, as is the case for RTK, or using a dynamic base, as is the case for DRTK. These algorithms are explained and results are shown in this paper.

The difficulty of the RTK algorithm is that it must resolve ambiguities in the carrier phase once the receiver has locked on to a satellite's signal. The least squares ambiguity decorrelation adjustment (LAMBDA) method was created to help resolve these ambiguities. When the baseline between GPS antennas is known, this known baseline can be used as a constraint and can be integrated into the LAMBDA method, resulting in a C-LAMBDA method. This research uses the UWB range measurements in place of the known baseline in the C-LAMBDA method and results comparing it to the LAMBDA method are presented.

By looking at the experimental results, some conclusions can be made. As long as the accuracy of the UWB range measurements is within a few centimeters, it is shown that it can be used in the C-LAMBDA method as the baseline constraint in helping to resolve the carrier phase ambiguities.

1. Introduction

The goal of this research is to successfully integrate the range measurements from UWB radios into the LAMBDA method of the RTK algorithm to improve the time-to-first-fix of the carrier phase integer ambiguities using lower costing single-frequency GPS receivers. This research has applications in scenarios where the relative positioning between vehicles is desired, such as a leader-follower scenario.

1.1 Background and Motivation

The accuracy of GPS is usually within a couple of meters. This is fine if a vehicle is driving down the road and needs to know which road it's on and where to turn, but some applications require much more accurate position solutions. There are several error sources that can degrade the GPS signal, but most of the error comes from the atmospheric refraction of the signal as it travels through the ionosphere and troposphere. Differential techniques can be used to mitigate these common errors to receivers that are within a close proximity, usually accepted to be under about 10 kilometers. This method of eliminating common errors is known as Differential GPS (DGPS). For applications that require an even more accurate position solution than DGPS, a technique called RTK positioning is an alternative.

One advantage that GPS has is that its errors are zero mean, so static base stations have a very accurate global position. Originally, the carrier phase of the signal was not used for positioning, but in the past few decades it has been actively researched for its obtainable millimeter-level accuracy. However, using the carrier phase for positioning requires solving the whole number of cycles the carrier phase measurement is off by from the receiver to each of the visible satellites. The floating point ambiguities can be easily estimated, but the ambiguities must be integers and are more difficult to calculate. Rounding these decimals might work on occasion, but it does not take into account the signal-to-noise ratio of each satellite's signal. After using a Kalman filter for the floating point estimation, the LAMBDA method was created by researchers at Delft University, namely Peter Teunnisen, to calculate the integer carrier phase ambiguities. Once these carrier phase ambiguities are resolved, an accuracy of around 2 centimeters can be achieved and maintained as long as the receiver stays locked on to each of the satellites.

RTK requires a static base station, which isn't always available, such as the instance of an autonomous helicopter landing on a moving aircraft carrier. For applications like this, the global positioning of the receiver's positions isn't as important as the relative positioning of the receivers to each other. An algorithm used to calculate the relative position vector (RPV) between the receivers is the Dynamic-base RTK (DRTK) technique. Any application that is more concerned with relative positioning versus global positioning could use this, such as a micro aerial vehicle (MAV), as found in [7, 12], or an autonomous unmanned aerial vehicle (UAV) refueling scenario, as found in [5].

Some scenarios have a fixed baseline between the GPS antennas, such as when antennas are mounted on top of a car or plane to determine the vehicle's attitude. These situations present an opportunity to use this fixed baseline as either a constraint or validation in the RTK algorithm, as shown in the work done in [2] and [3]. This method is called the Constrained-LAMBDA (C-LAMBDA) method and was initially researched by Teunnisen. It works especially well on ships and planes, which allow for longer antenna baseline distances.

The C-LAMBDA algorithm works for fixed baseline because the antenna baselines are precisely known. Alternatively, this research investigates the use of a dynamic, non-exact baseline, such as the range measurements from ultra-wideband radios. UWBs can calculate the range between two antennas to within a centimeter by sending a pulse on a wide range of frequencies down and back between antennas and calculating the two way time-of-flight. Because the carrier wavelength is approximately 19 centimeters, the accuracy of the UWB is critical if it is to be implemented in the C-LAMBDA method.

1.2 Previous Works

Most of this research revolves around the details of resolving the carrier phase ambiguities. This has been an area of active research due to its implications for relative positioning and navigation. The LAMBDA algorithm was developed by Peter Teunnisen in the mid-1990s as a way to resolve this carrier phase ambiguity to achieve centimeter-level accuracy. Since then, the research as adjusted to using constraints in the LAMBDA method to resolving these ambiguities on lower quality single-frequency receivers.

Several efforts have been made to incorporate the *a priori* knowledge of the baseline between a pair of antennas to improve the accuracy in finding the carrier phase ambiguities. When done with three GPS antennas mounted on a vehicle, the vehicle's attitude can be determined. There are two main search strategies that were developed to find these ambiguities, the search and shrink approach, as shown in the research done in [6], or the search and expand approach, as shown in the research done in [12].

Due to the ability of the UWB to reject multi-path, it has become a valued asset in the GPS field. Stanford University and the University of Calgary have both shown the UWB measurements can be tightly-coupled with GPS measurements in a Kalman filter to improve DGPS positions, as shown in [11] and [9]. The University of Calgary also recently implemented an extended Kalman filter in which differential GPS pseudorange, Doppler and carrier phase measurements were used in conjunction with UWB range measurements between a vehicle and two points on either side of the road. Their results indicated that the GPS and UWB integrated positioning system could significantly improve the GSP float solution and carrier phase ambiguity resolution compared to the scenario using only GPS measurements. This was shown in [8].

At the Department of Geomatics Engineering at the University of Calgary, higher quality UWB radio measurements were integrated into the RTK algorithm to help resolve the carrier phase ambiguities in surveying-type scenarios. The UWB ranges were specifically used to improve the carrier phase float convergence time, which led to faster computation times in resolving the integer ambiguities, as shown in the work done in [10].

2. Methods

A highly accurate relative position vector can be achieved between two GPS receivers that are within a close proximity. This is called a RTK system, which requires that one of the GPS receivers be constrained to a known location. For scenarios where a static base station can't be used, the DRTK method can be used to give the highly accurate RPV between receivers, although the highly accurate global position from the RTK is no longer achieved. When the baseline between GPS antennas is precisely known, this *a priori* baseline measurement can be used as a constraint in the LAMBDA method of the RTK algorithm. Ultra-wideband radios can give range measurements that are centimeter-level accurate. This research will use these UWB range measurements in the C-LAMBDA method as the *a priori* baseline measurements to allow for a dynamic baseline between GPS receivers.

2.1 Dynamic-base RTK

When two GPS receivers are within close proximity to each other, most of the errors that degrade the GPS signal are common to both receivers. Most of this error common to both receivers comes from the signal propagating through the atmosphere. The relative position between two GPS receivers can be estimated with centimeter-level accuracy by exploiting the accuracy of the carrier phase measurement. The carrier phase measurement and what it's comprised of is shown in Equation (1), where the carrier phase is in meters.

$$\phi_{R_1}^{S_1} = ||r_{R_1}^{S_1}|| + c(\delta t_{R_1} - \delta t^{S_1}) + \lambda(T^{S_1} - I^{S_1} + N^{S_1}) + M_{\phi_{R_1}} + v_{\phi_{R_1}}^{S_1} \quad (1)$$

This equation shows the carrier phase contains the true range, the clock errors of the satellite and receiver, the ionospheric and tropospheric errors, multi-path, residual noise, and the integer ambiguities.

The atmospheric errors in the carrier phase measurement are highly correlated between GPS receivers that are within a close proximity. The satellite clock error that is experienced by the receivers to the same satellite will be nearly identical. Because of this, these errors can be differenced out in a procedure called single differencing, in which the carrier phase measurements from two different receivers to common satellites are subtracted from each other. This results in Equation (2).

$$\Delta\phi_{R_1-R_2}^{S_1} = \rho_{R_1-R_2}^{S_1} + c\delta t_{R_1-R_2} + \lambda N_{R_1-R_2}^{S_1} + v_{\phi_{R_1-R_2}}^{S_1} \quad (2)$$

Note that now the atmospheric and satellite clock errors have no been eliminated. The multi-path error has been eliminated, though this is an assumption as the multi-path error is dependent on the GPS receiver's local environment.

This single differenced carrier phase measurement still contains the receiver clock errors, the integer ambiguities, and the residual noise error. The receiver clock errors can be eliminated in a procedure called double differencing, in which the single differenced carrier phase measurements are subtracted from a base satellite, usually chosen as the highest satellite in the sky. The resulting double differenced carrier phase measurement is shown in Equation (3).

$$\nabla\Delta\phi_{R_1-R_2}^{S_1-S_2} = r_{R_1-R_2}^{S_1-S_2} + v_{R_1-R_2}^{S_1-S_2} + \lambda N_{R_1-R_2}^{S_1-S_2} \quad (3)$$

The double differenced carrier phase measurements now contain only the true range and the integer ambiguities, along with some residual noise. It should be noted that the residual noise is amplified by both of these differencing procedures.

Before the relative position vector can be calculated, the carrier phase integer ambiguity must be estimated and removed from the equation. The double differenced integer ambiguities are first estimated as a floating point using a Kalman filter. Because we know that the ambiguities are integers, the floating point ambiguities are then converted to integers using the LAMBDA method. Once the integer ambiguities are estimated, they are subtracted from the double differenced carrier phase measurements. The RPV between GPS receivers can now be calculated via a least squares solution. For more on this integer ambiguity estimation process, refer to the work done in [1].

2.2 Baseline-Constrained LAMBDA

With the *a priori* knowledge of the baseline between GPS receivers, the search space in the LAMBDA method can be modified to incorporate this baseline length. This is accomplished by creating an auxiliary search space that includes the knowledge of the baseline length. For more information on the carrier phase ambiguity search space that the LAMBDA method creates, see the research done in [13].

The minimization equation of the standard LAMBDA method is shown in Equation (4), and the minimization equation for the baseline-constrained LAMBDA, which now uses the *a priori* known baseline length, is shown in Equation (5).

$$\check{a} = \min_{a \epsilon Z^{Nn}} ||\hat{a} - a||_{Q_{\hat{a}}}^2 \quad (4)$$

$$\check{a} = \min_{a \epsilon Z^{Nn},\ b \epsilon R^3} (||\hat{a} - a||_{Q_{\hat{a}}}^2 + ||\check{b}_l(a) - \hat{b}(a)||_{Q_{\hat{b}_l}}^2) \quad (5)$$

The term on the right of Equation (5) contains the fixed baseline solution calculated when constraining the integer ambiguities to the baseline length. It is found by solving a quadratically constrained least squares problem. For a more detailed look at the derivation and solving of these minimization problems, see the work in [my thesis] and [GNSS compass].

Now that the baseline term is in the minimization problem, the search space is no longer elliptical, so the LAMBDA method can no longer be used. However, there are two search strategies that can be used to find the integer ambiguities, the search and shrink approach, or the search and expand approach. For this research, the search and expand approach was used, although both search strategies should result in the same integer ambiguities. For more information on the search and expand approach, see the work done in [12].

The LAMBDA method returns the top two candidates for the integer ambiguities, and if the ratio test is passed, the top candidate is deemed the correct ambiguities. However, the C-LAMBDA method only returns one candidate, so the ratio test cannot be performed. In [12], a probability based confidence level was developed to determine the integrity of the carrier phase integer ambiguities.

2.3 UWB Radios

Though GPS is a relatively reliable positioning system, there are several limitations to its use. For instance, GPS receivers require good sky visibility to be able to receiver the GPS signals. This requirement may be severely hampered in applications that are surrounded by trees, buildings, or even indoor applications. Ultra-wideband radios (UWBs) can be used in these situations of GPS denied environments to help with real time localization with stationary or dynamic points of reference. This type of technology could be used in many different applications, such as: autonomous robot swarm scenarios, relative localization in GPS denied environments, or to help alleviate GPS multipath errors and IMU drift through precise differential ranging.

Whereas most radios transmit signals centered at a specific sinusoidal frequency, UWBs send out short impulses over a broad range of frequencies. By using this short duration, pulsed RF technology, the highest possible bandwidths and lowest possible center frequencies can be achieved. This technology can be used for communicating, radar, or ranging and locating applications. For more information on UWB technology, see [15].

The UWBs can calculate the peer-to-peer distance between radio transceivers using the two way time of flight (TW-TOF) technique. This technique is highly accurate and has a wide variety of applications. This TW-TOF range measurement equation is seen in Equation (6).

$$TOF = \frac{1}{2}(CT - TAT - LED_1 - LED_2) \qquad (6)$$

This equation shows that the TOF is calculated based on the conversation time (CT), the turn around time (TAT), and both the leading edge detections (LED). Once the TOF has been calculated, it is multiplied by the speed of light to get a range measurement. These range measurements are centimeter-level accurate. For more information on the range measurement calculation, see [4]. For experimental results of the UWB radios, see the work done in [1].

3. Experimental Results

The algorithms described earlier were used in post-process using the data collected experimentally at Auburn University. For the experiments performed, two dual-frequency NovAtel Propak GPS receivers with NovAtel Pinwheel antennas were used in conjunction with two PulsON 400 UWB radios that were donated from Time Domain. They were placed at two different separation distances, both UWBs having a good line of sight to each other and with both GPS antennas having good sky visibility. Though the receivers were dual-frequency, for this work only the L1 frequency was analyzed as an attempt to show the advantage of the C-LAMBDA method with lower costing setups.

3.1 Zero Baseline C-LAMBDA

For this experiment, two NovAtel Propak GPS receivers were used with a single NovAtel Pinwheel antenna whose signal was split to both receivers. Zero baseline tests are good for testing because the single and double differencing procedures should eliminate nearly all the errors because the receivers are hooked up to the same antenna and will get the same errors.

The data was collected and run post-process through the C-LAMBDA method, which uses the *a priori* knowledge of the zero baseline and is calculated epoch-by-epoch. The results of the fixed RTK solution versus the C-LAMBDA solution are shown in Figure 1.

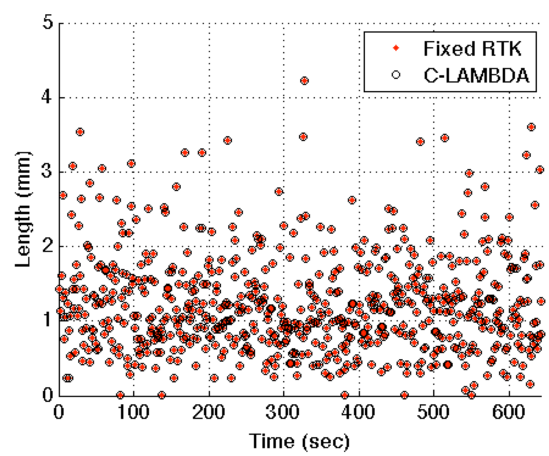

Figure 1: Zero baseline fixed RTK and C-LAMBDA

As can be seen, the C-LAMBDA method is able to resolve the carrier phase integer ambiguities for each epoch of the experiment.

The integer ambiguity validation, as previously mentioned, gives a probability that the calculated integer ambiguities from the C-LAMBDA method are the correct integer ambiguities; the results from this validation method shown in Figure 2. As can be seen, the probability that correct fixed were calculated from the C-LAMBDA method for the zero baseline experiment start out and maintain near 100% confidence with an average of 97.9% over the entire run, which shows extreme confidence that the correct fixes were calculated.

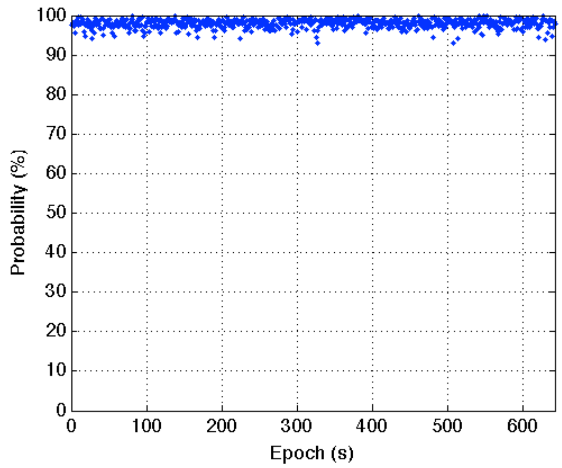

Figure 2: Zero baseline C-LAMBDA integer ambiguity validation

Because the C-LAMBDA method uses the *a priori* knowledge of the baseline between GPS antennas, the baseline is assumed to be correct. However, there will usually always be some sort of inherent error in this baseline depending on the application. If the antennas are mounted on the wings of an aircraft, this error could come from the flexing of the wings. In this research, the error will come from the UWB range measurements.

To test the robustness of the C-LAMBDA method when it comes to error in the *a priori* baseline measurement, the experiment was run with an injected error in the baseline that is sent to the C-LAMBDA method. With the knowledge of the correct integer ambiguity fixes, the percentage of the total correct integer ambiguity fixes that the C-LAMBDA method with the erroneous baseline calculates is determined.

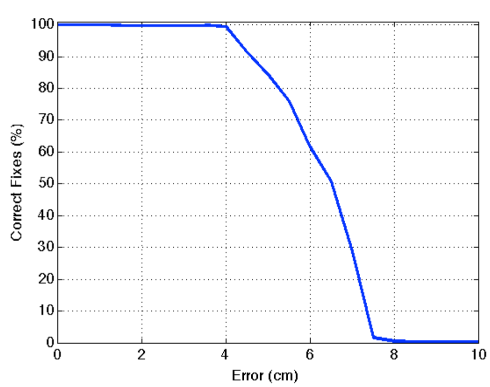

Figure 3: Zero baseline *a priori* error in C-LAMBDA method

The results of this robustness test of the C-LAMBDA method are shown in Figure 3. As can be seen, the C-LAMBDA method is able to withstand a 4 centimeter error before it starts giving faulty integer ambiguity fixes. Once the error in the *a priori* baseline reaches 8 centimeters, the C-LAMBDA method is no longer able to compute the correct integer ambiguities for even one epoch in the entire data set.

3.2 Short Baseline C-LAMBDA with UWBs

A short baseline of approximately 20 meters was used between GPS antennas, which then the UWB radio antennas were placed parallel to the GPS antennas to give the same baseline. The UWB range measurements were run through the C-LAMBDA method as the *a priori* baseline, and the results are shown in Figure 4.

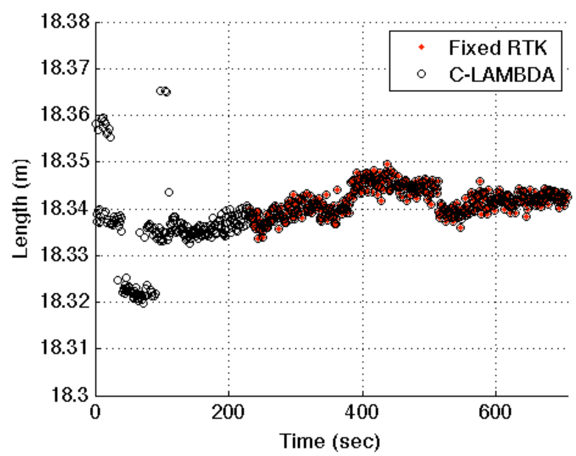

Figure 4: Short baseline C-LAMBDA method results using UWB range measurements

Though jumpy at first, the C-LAMBDA is eventually able to lock on to the correct integer ambiguities and maintain lock for the rest of the experiment.

The results from the integer ambiguity validation check are shown in Figure 5.

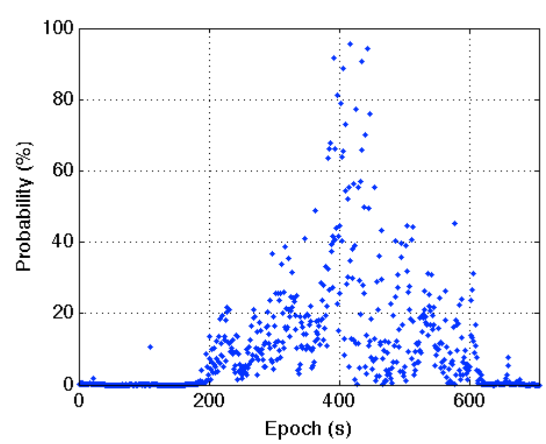

Figure 5: Short baseline C-LAMBDA integer ambiguity validation

Based on the results, the algorithm is not nearly as confident with the C-LAMBDA method in the short baseline as it was in the zero baseline test. Even though it takes the standard LAMBDA approximately 250 seconds to lock on to the correct integer ambiguities, it takes the C-LAMBDA method nearly 400 seconds before it is confident that the correct integer ambiguities have been calculated.

3.3 Long Baseline C-LAMBDA with UWBs

For the longer baseline test, the antennas were placed approximately 70 meters apart, with good line of sight between the UWB radio antennas and good sky visibility for the GPS antennas. The results from the C-LAMBDA method using the UWB range measurements as the *a priori* baseline measurement are shown in Figure 6. Because the L1 frequency alone couldn't resolve the unknown carrier phase integer ambiguities, the results from the RTK that used both the L1 and L2 frequencies are shown to give a perspective on the level of accuracy of the solutions from the C-LAMBDA method, which used only the L1 frequency.

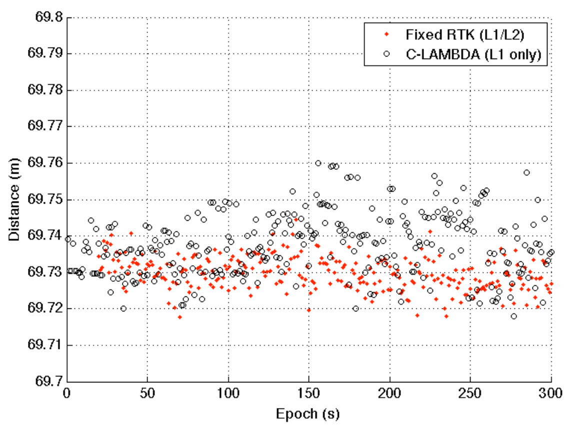

Figure 6: Long baseline single-frequency C-LAMBDA versus dual-frequency fixed RTK

This figure shows that the C-LAMBDA method does give relative distances that are very close to the standard LAMBDA method solutions using both the L1 and L2 frequencies. This is due to the baseline constraint that looks for integer ambiguity candidates that give an RPV that is close to the *a priori* baseline. However, just because the magnitude of the RPV is close to the *a priori* baseline does not guarantee that the x, y and z components of the RPV are close to the actual components.

A plot of the y and z components of the RPV between GPS receivers is shown in Figure 7. In this plot, the y and z components using the C-LAMBDA method are shown with the y and z components of the RPV from both the float and fixed RTK solutions using the standard LAMBDA method.

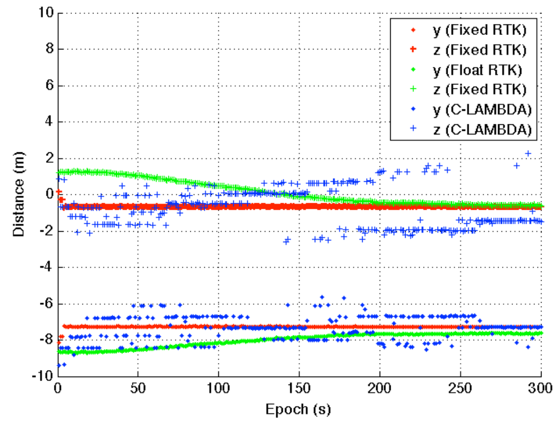

Figure 7: Long baseline C-LAMBDA versus float and fixed RTK RPV components

As can be seen, the y and z components from the float RTK solution converge towards those from the fixed RTK solution, but the y and z components from the C-LAMBDA method never converge towards anything and can be more than 2 meters off the fixed RTK components. So though Figure 6 makes it appear that the C-LAMBDA method is about as accurate as the standard LAMBDA, Figure 7 shows that the C-LAMBDA method gives erroneous vector components that equal to a similar magnitude that the fixed RTK solution gives.

4. Conclusions

The C-LAMBDA method was shown to be able to handle about a 4 centimeter error in the *a priori* baseline measurement and still give the correct carrier phase integer ambiguities. With the accuracy of the UWB range measurements being less than 2 centimeters, they should be able to suffice as the *a priori* baseline measurement. However, determining if the correct integer ambiguities were resolved is not as definitive as the ratio test in the standard LAMBDA method. The integer ambiguity validation method of the C-LAMBDA has shown to resolve the wrong integer ambiguities, and also shown little confidence even when the correct integer ambiguities were found.

While the C-LAMBDA method did show that it could resolve the correct integer ambiguities with both a zero baseline and a baseline of about 20 meters, the C-LAMBDA method did not appear to work as well at longer baseline upwards of 70 meters. Though the magnitude of the RPV from the C-LAMBDA method is very close to the dual-frequency RTK solution, the x, y and z vector components of the RPV between receivers can be several meters off the actual distances.

In conclusion, the UWB range measurements are accurate to be used as the *a priori* baseline measurement in the C-LAMBDA method; however, the C-LAMBDA method does not work as well at longer baselines and it is more difficult to determine if the correct integer ambiguities have been chosen when compared to the ratio test in the standard LAMBDA method.

4.1 Future Work

There are several further steps to be taken with this research. The UWB radios could be programmed specifically for GPS applications. The trade-off with UWBs is that you can increase the accuracy at the expense of decreasing the update rate. For most non-GPS applications, the accuracy isn't as important as the faster update rates. But with GPS receivers commonly logged at a 1 Hz update rate, the accuracy of the UWB range measurements can be significantly improved while still maintaining the required 1 Hz update rate of the GPS receivers. With this higher accuracy from the UWBs, the C-LAMBDA method should be able to perform more smoothly as the *a priori* baseline measurement will be more accurate and precise.

This research showed that the C-LAMBDA method worked well at baselines less than 20 meters, but had difficulty fixing the integer ambiguities at longer baselines. To examine where exactly the C-LAMBDA method starts to degrade, multiple static datasets could be taken at increasingly longer baseline intervals. Once the C-LAMBDA method starts fixing wrong integers, the search space could be increased to search for more integer combinations. Also, the ratio test in the standard LAMBDA method took longer to pass, but would always lock on to the correct integer ambiguities once the ratio was above the ratio tolerance. However, the probability based confidence test of the C-LAMBDA method would sometimes allow erroneous integer ambiguity combinations and would sometimes not allow the correct integer ambiguity combinations. This could be researched further, as well as developing another way to test the integer ambiguities that result from the C-LAMBDA method.

The work only implemented the L1 frequency observation measurements to appeal to lower costing setups and applications. However, the L2 frequency observation measurements could also be added in the C-LAMBDA method to examine how the C-LAMBDA method works using single-frequency versus dual-frequency. Also, the search and expand approach was used in the C-LAMBDA method in this research, so alternatively the search and shrink approach could be examined to see if that allows for the C-LAMBDA to work at longer baselines.

An additional issue that would need to be resolved to integrate the UWB range measurements successfully into the C-LAMBDA method is the timing of both the devices. The GPS has a pulse-per-second (PPS) signal that the UWBs could be synchronized with. Because the timing of the UWBs and GPS receivers are not perfectly aligned, only static data could be analyzed in this research. If the timing of the UWBs and GPS receivers were synchronized, then dynamic tests could be done and the data utilizing the C-LAMBDA method could be analyzed.

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